

Let X_n , $n \geq 0$, be a submartingale with $\sup_{n \geq 0} X_n < \infty$. Let $\xi_n = X_n - X_{n-1}$, $n \geq 1$, and suppose $\mathbb{E}(\sup_{n \geq 1} \xi_n^+) < \infty$. Show that X_n converges a.s.