11th Homework Set — Solutions

Chapter 8

Problem 8.4 (a) $P(\sum_{i=1}^{20} X_i > 15) \leq \frac{20}{15}$.

(b)

$$\begin{aligned} P(\sum_{i=1}^{20} X_i > 15) &= P(\sum_{i=1}^{20} X_i > 15.5) \approx P(Z > \frac{15.5 - 20}{\sqrt{20}} \\ &= P(Z > -1.006) \approx .8428. \end{aligned}$$

Problem 8.5 Let X_i be the i-th round-off error, then

$$E(\sum_{i=1}^{50} X_i) = 0, \quad \text{Var}(\sum_{i=1}^{50} X_i) = \frac{50}{12}.$$

Hence by the central limit theorem

$$P\left(|\sum_{i=1}^{50} X_i| > 3\right) \approx P(|Z| > \frac{3}{\sqrt{12/50}}) = 2P(Z > 1.47) = .1416.$$

Problem 8.7 If we let X_i be the lifetime of the i-th light bulb, then the desired probability is

$$P\left(\sum_{i=1}^{100} X_i > 525\right).$$

It follows from the central limit theorem that $\sum_{i=1}^{100} X_i$ is approximately a normal random variable with mean 500 and variance 2500. Consequently the desired probability is equal to

$$P(Z > \frac{525 - 500}{50}) = P(Z > .05) = .3085.$$

Problem 8.8 If we let X_i be the lifetime of the i-th light bulb and R_i be the time to replace the i-th light bulb, then the desired probability is

$$P\left(\sum_{i=1}^{100} X_i + \sum_{i=1}^{99} R_i \leq 550\right).$$

It follows from the central limit theorem that $\sum_{i=1}^{100} X_i$ is approximately a normal random variable with mean 500 and variance 2500 and that $\sum_{i=1}^{99} R_i$ is approximately a normal random variable with mean 24.75 and variance 99/48, therefore $\sum_{i=1}^{100} X_i + \sum_{i=1}^{99} R_i$ is approximately a normal random variable with mean 524.75 and variance 2502.02. Consequently the desired probability is equal to

$$P(Z \leq \frac{550 - 524.75}{\sqrt{2502.02}}) = P(Z \leq .505) = .693.$$

Problem 8.15 For $i = 1, \ldots, 10000$, let X_i the claim amount of the i-th policyholder. Then $E[X_i] = 240$ and $\mathrm{Var}(X_i) = 800^2$. Thus

$$\begin{aligned}
&P(\sum_{i=1}^{10000} X_i > 2700000) \\
=&P\left(\frac{\sum_{i=1}^{10000} X_i - 2400000}{80000} > \frac{2700000 - 2400000}{80000}\right) \\
\approx&1 - \Phi(3.75) \approx 0.
\end{aligned}$$