Math 461, Section C13, Spring 2024

Renming Song

University of Illinois Urbana-Champaign

January 17, 2024

Outline

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Course syllabus is available from my homepage:
https://rsong.web.illinois.edu/461s24/461s24.html

Textbook: Sheldon Ross, A First Course in Probability, 9th Edition, 2014, Pearson.

You do need a copy of this book. Homework will be assigned from this book. You need to make sure your are doing the right problems.

Office Hours: MWF: noon-12:50 pm in 227 CAB.

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The basic materials of this course are not too difficult. But each problem requires thinking. In this sense, this course will not be easy. If you need help during the semester, please get help in a timely manner. Do not wait until the day before the test to get help.

Homework problems will be assigned daily. I will post the assigned exercises on the my home page. Homework problems will be collected weekly on Fridays and 4 or 5 randomly selected problems will be graded. Late homework will not be graded and credited. The two lowest scores on the homework assignments will be dropped.

There will be 2 tests. These tests will be during regular class times. The dates are: Test 1: Friday, March 01; Test 2: Friday, April 12. The final will be Friday, May 10, 1:30 pm to 4:30 pm.

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The tests and the final will be in the regular classroom and will be closed book. No cheat sheet is allowed.

There is no makeup for the tests or the final except for medical reasons, in which case you have to provide medical documents from the doctor.

Each test accounts for 25% of the grade, the final accounts for 40% of the grade and the homework accounts for 10% of the grade.

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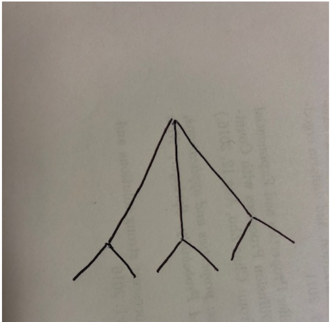
1. Course Info
2. **Multiplication Rule**
3. Permutations
4. Combinations

The basic principle of counting, or simply, the multiplication rule, is very important for this course. It tells us how to count things.

Multiplication Rule

Suppose that 2 experiments are to be performed. If experiment 1 can result in m possible outcomes, and if for each possible outcome of experiment 1, there are n possible outcomes for experiment 2, then all together there are mn possible outcomes for these 2 experiments.



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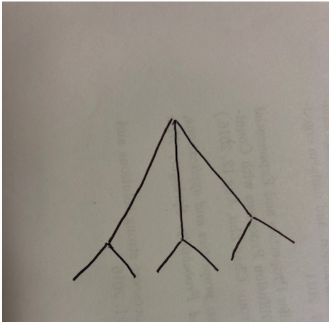

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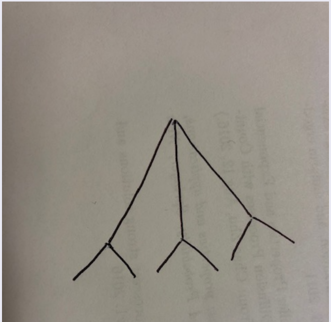

The Multiplication Rule can be easily generalized to r experiments.

Multiplication Rule for r Experiments

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Example 1

How many different 6-place license plates are possible if the first 3 places are to be occupied by letters (the English alphabet) and the last 3 by numbers (0, 1, . . . , 9)? How many would be possible if repetition are prohibited?

Answer: (a) $26^3 \cdot 10^3$; (b) $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8$.

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Example 2

A woman wants to give her son 14 different baseball cards within a 7-day period. (All 14 cards are to be given out during these 7 days.) If she gives her son no more than once per day, in how many ways can this be done?

Answer: 7^{14}.

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Suppose that we have n distinct objects, we can put them in different ordered arrangements. Each of these ordered arrangement is known as a *permutation*.

By the multiplication rule, there are $n! := 1 \cdot 2 \cdot \cdots \cdot n$ permutations of n distinct objects.

Convention: $0! = 1$.

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Example 3

John has 12 books that he is going to put on his bookshelf. Of these, 4 are fictions, 3 are physics books, 3 are math books and 2 are chemistry books. John wants to put his books on the shelf so that all books of the same subject are together on the shelf. In how many ways can he arrange his books?

Answer: $4! \cdot 4! \cdot 3! \cdot 3! \cdot 2!$.

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Example 4

In how many ways can 8 people be seated in a row if (a) there is no restriction? (b) person A and person B must sit together? (c) there are 4 men and 4 women, and no 2 men or 2 women can sit next to each other? (d) there are 5 men (and 3 women) and the men must sit together? (e) there are 4 married couples and each couple must sit together?

Answer (a) $8!$; (b) $7! \cdot 2!$; (c) $2 \cdot 4! \cdot 4!$; (d) $4! \cdot 5!$; (e) $4! \cdot 2^4$.

Example 4

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We now determine the number of permutations of a set of n objects when some objects are indistinguishable from each other.

How many different letter arrangements can be formed using the letter PEPPER?

Answer:

$$\frac{6!}{3! \cdot 2!}$$

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In general, there are

$$\frac{n!}{n_1! \cdot n_2! \cdots n_r!}$$

different permutations of n objects, of which n_1 are alike (indistinguishable), n_2 are alike, ..., n_r are alike.

Example 5

David has 10 blocks, 4 are orange, 3 are blue and 3 are red. How many arrangements are possible if blocks of the same color are indistinguishable?

Answer:

$$\frac{10!}{4! \cdot 3! \cdot 3!}.$$

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We are often interested in determining the number of different groups of *r* objects (that is, the order in which members of the group are chosen is irrelevant) that can chosen from a total of *n* distinct objects.

For example how many different groups of 5 cards can be formed from an ordinary deck of 52 cards?

Answer:

$$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!} = \frac{52!}{5! \cdot 47!}.$$

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In general $n(n-1)\cdots(n-r+1)$ represents the number of different ways that r items can be selected from n distinct items if the order *were* relevant. Since each group of r items will be counted $r!$ times, the number of different groups of r items that can be chosen from n distinct items is

$$\frac{n(n-1)\cdots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}.$$

Notation:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

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$$\binom{n}{0} = 1, \quad \binom{n}{n} = 1.$$

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A committee of 4 is to be formed from group of 10 people? How many different ways can the committee be chosen?

Answer: $\binom{10}{4}$.

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Example 7

From a group of 5 women and 7 men, how many different committees of 5, consisting of 2 women and 3 men, can be formed? What if 2 of the men are feuding and refuse to serve together?

Answer: (a)

$$\binom{5}{2} \cdot \binom{7}{3};$$

(b)

$$\binom{5}{2} \cdot \left(\binom{5}{3} + \binom{2}{1} \cdot \binom{5}{2} \right) = \binom{5}{2} \cdot \left(\binom{7}{3} - \binom{5}{1} \right)$$

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